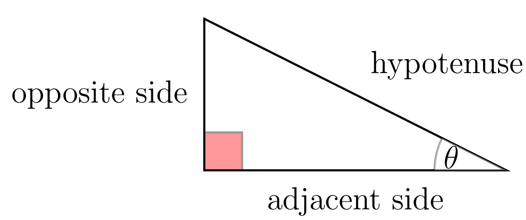
| **Facts about Trigonometric Substitution** | **Explanation** |
|---|---|
| How is Trigonometric Substitution Used? | Trigonometric Substitution (Trig. Sub.) helps simplify a radical or an expression. It is useful if the integrand contains a sum or a difference of squares, often under a radical. |
| Expression — Trig. Sub. — Identity<br>$\sqrt{a^2 - x^2}$ — $x = asin(\theta)$ — $sin^2(\theta) + cos^2(\theta) = 1$<br>$\sqrt{x^2 - a^2}$ — $x = asec(\theta)$ — $tan^2(\theta) + 1 = sec^2(\theta)$<br>$\sqrt{x^2 + a^2}$ — $x = atan(\theta)$ — $tan^2(\theta) + 1 = sec^2(\theta)$ | A summary of trigonometric substitutions in which the expression considered, trigonometric substitution used, and pythagorean identity applied are in the left column, middle column, and right column respectively. |
| Review of the Relationship between Trigonometric Functions and Right Triangles<br> | $sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$<br>$csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite side}}$<br>$cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$<br>$sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent side}}$<br>$tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$<br>$cot(\theta) = \frac{\text{adjacent side}}{\text{opposite side}}$ |

1. We will be applying Trigonometric Substitution in order to evaluate $\int \frac{1}{\sqrt{1-x^2}}\ dx$ through the following parts.

   (a) Determine the appropriate Trigonometric Substitution to use out of the following: $x = asin(\theta), x = atan(\theta)$, or $x = asec(\theta)$. Afterwards, compute $dx$.

   (b) Substitute $x$ and $dx$ and simplify as much as possible, using trigonometric identities to get rid of the square root.

   (c) Evaluate the integral and change your answer back in terms of $x$. Could you have evaluated your integral in an easier way?

2. We will be applying Trigonometric Substitution in order to evaluate $\int \frac{1}{\sqrt{4+x^2}}\ dx$ through the following parts.

   (a) Determine the appropriate Trigonometric Substitution to use out of the following: $x = asin(\theta), x = atan(\theta)$, or $x = asec(\theta)$. Afterwards, compute $dx$.

   (b) Substitute $x$ and $dx$ and simplify as much as possible, using trigonometric identities to get rid of the square root. Note that $\int sec(\theta)d\theta = ln|sec(\theta) + tan(\theta)|$+C.

   (c) Evaluate the integral and change your answer back in terms of $x$. This may require the use of a right triangle.

3. We will be applying Trigonometric Substitution in order to evaluate $\int \frac{\sqrt{x^2 - 3}}{x}\, dx$ through the following parts.

   (a) Determine the appropriate Trigonometric Substitution to use out of the following: $x = asin(\theta), x = atan(\theta)$, or $x = asec(\theta)$. Afterwards, compute $dx$.

   (b) Substitute $x$ and $dx$ and simplify as much as possible, using trigonometric identities to get rid of the square root.

   (c) Evaluate the integral and change your answer back in terms of $x$. This may require the use of a right triangle.